

# Mathematica 11.3 Integration Test Results

Test results for the 52 problems in "4.4.0 (a  $\operatorname{trg}$ ) $^m$  (b  $\cot$ ) $^n$ .m"

Problem 39: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b \cot[e + f x])^n (a \sin[e + f x])^m dx$$

Optimal (type 5, 87 leaves, 2 steps):

$$-\frac{1}{b f (1+n)} (b \cot[e + f x])^{1+n} \text{Hypergeometric2F1}\left[\frac{1+n}{2}, \frac{1}{2} (1-m+n), \frac{3+n}{2}, \cos[e + f x]^2\right] \\ (a \sin[e + f x])^m (\sin[e + f x]^2)^{\frac{1}{2} (1-m+n)}$$

Result (type 6, 2957 leaves):

$$\left( 2 (3+m-n) \text{AppellF1}\left[\frac{1}{2} (1+m-n), -n, 1+m, \frac{1}{2} (3+m-n), \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \cos\left[\frac{1}{2} (e+f x)\right]^2 \right. \right. \\ \left. \left. \cot\left[\frac{1}{2} (e+f x)\right] \cot[e+f x]^n (b \cot[e+f x])^n \sin[e+f x]^m (a \sin[e+f x])^m \right) \right/ \\ \left( f (1+m-n) \left( -2 n \text{AppellF1}\left[\frac{1}{2} (3+m-n), 1-n, 1+m, \frac{1}{2} (5+m-n), \tan\left[\frac{1}{2} (e+f x)\right]^2, \right. \right. \right. \right. \\ \left. \left. \left. -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] - 2 (1+m) \text{AppellF1}\left[\frac{1}{2} (3+m-n), -n, 2+m, \frac{1}{2} (5+m-n), \right. \right. \\ \left. \left. \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] + (3+m-n) \text{AppellF1}\left[\frac{1}{2} (1+m-n), -n, \right. \right. \\ \left. \left. 1+m, \frac{1}{2} (3+m-n), \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \cot\left[\frac{1}{2} (e+f x)\right]^2 \right) \right. \\ \left( - \left( 2 (3+m-n) n \text{AppellF1}\left[\frac{1}{2} (1+m-n), -n, 1+m, \frac{1}{2} (3+m-n), \right. \right. \right. \\ \left. \left. \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] \right. \\ \left. \left. \cos\left[\frac{1}{2} (e+f x)\right]^2 \cot\left[\frac{1}{2} (e+f x)\right] \cot[e+f x]^{-1+n} \sin[e+f x]^{-2+m} \right) \right/ \\ \left( (1+m-n) \left( -2 n \text{AppellF1}\left[\frac{1}{2} (3+m-n), 1-n, 1+m, \frac{1}{2} (5+m-n), \tan\left[\frac{1}{2} (e+f x)\right]^2, \right. \right. \right. \right. \\ \left. \left. \left. -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] - 2 (1+m) \text{AppellF1}\left[\frac{1}{2} (3+m-n), -n, 2+m, \frac{1}{2} (5+m-n), \right. \right. \\ \left. \left. \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \right] + (3+m-n) \text{AppellF1}\left[\frac{1}{2} (1+m-n), -n, \right. \right. \right. \\ \left. \left. \left. 1+m, \frac{1}{2} (3+m-n), \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \cot\left[\frac{1}{2} (e+f x)\right]^2 \right) \right)$$



$$\begin{aligned}
& \frac{1}{3+m-n} (1+m) (1+m-n) \operatorname{AppellF1}\left[1 + \frac{1}{2} (1+m-n), -n, 2+m, 1 + \frac{1}{2} (3+m-n), \right. \\
& \quad \left. \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right]\right] \Bigg) / \\
& \left( (1+m-n) \left(-2 n \operatorname{AppellF1}\left[\frac{1}{2} (3+m-n), 1-n, 1+m, \frac{1}{2} (5+m-n), \tan\left[\frac{1}{2} (e+f x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] - 2 (1+m) \operatorname{AppellF1}\left[\frac{1}{2} (3+m-n), -n, 2+m, \frac{1}{2} (5+m-n), \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] + (3+m-n) \operatorname{AppellF1}\left[\frac{1}{2} (1+m-n), -n, \right. \right. \\
& \quad \left. \left. 1+m, \frac{1}{2} (3+m-n), \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2 \cot\left[\frac{1}{2} (e+f x)\right]^2\right] \right) - \\
& \left( 2 (3+m-n) \operatorname{AppellF1}\left[\frac{1}{2} (1+m-n), -n, 1+m, \frac{1}{2} (3+m-n), \tan\left[\frac{1}{2} (e+f x)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] \cos\left[\frac{1}{2} (e+f x)\right]^2 \cot\left[\frac{1}{2} (e+f x)\right] \cot[e+f x]^n \sin[e+f x]^m \right. \\
& \quad \left. \left( - (3+m-n) \operatorname{AppellF1}\left[\frac{1}{2} (1+m-n), -n, 1+m, \frac{1}{2} (3+m-n), \tan\left[\frac{1}{2} (e+f x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] \cot\left[\frac{1}{2} (e+f x)\right] \csc\left[\frac{1}{2} (e+f x)\right]^2 + (3+m-n) \cot\left[\frac{1}{2} (e+f x)\right]^2 \right. \right. \\
& \quad \left. \left. \left( -\frac{1}{3+m-n} (1+m-n) n \operatorname{AppellF1}\left[1 + \frac{1}{2} (1+m-n), 1-n, 1+m, 1 + \frac{1}{2} (3+m-n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right] - \right. \right. \\
& \quad \left. \left. \frac{1}{3+m-n} (1+m) (1+m-n) \operatorname{AppellF1}\left[1 + \frac{1}{2} (1+m-n), -n, 2+m, 1 + \frac{1}{2} (3+m-n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right]\right) - \right. \\
& 2 n \left( -\frac{1}{5+m-n} (1+m) (3+m-n) \operatorname{AppellF1}\left[1 + \frac{1}{2} (3+m-n), 1-n, 2+m, 1 + \frac{1}{2} (5+m-n), \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right] + \right. \\
& \quad \left. \frac{1}{5+m-n} (1-n) (3+m-n) \operatorname{AppellF1}\left[1 + \frac{1}{2} (3+m-n), 2-n, 1+m, 1 + \frac{1}{2} (5+m-n), \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right]\right) - \\
& 2 (1+m) \left( -\frac{1}{5+m-n} (3+m-n) n \operatorname{AppellF1}\left[1 + \frac{1}{2} (3+m-n), 1-n, 2+m, 1 + \frac{1}{2} (5+m-n), \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right] - \right. \\
& \quad \left. \frac{1}{5+m-n} (2+m) (3+m-n) \operatorname{AppellF1}\left[1 + \frac{1}{2} (3+m-n), -n, 3+m, 1 + \frac{1}{2} (5+m-n), \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] \sec\left[\frac{1}{2} (e+f x)\right]^2 \tan\left[\frac{1}{2} (e+f x)\right]\right) \Bigg) / \\
& \left( (1+m-n) \left(-2 n \operatorname{AppellF1}\left[\frac{1}{2} (3+m-n), 1-n, 1+m, \frac{1}{2} (5+m-n), \tan\left[\frac{1}{2} (e+f x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] \right) \right)
\end{aligned}$$

$$\begin{aligned} & -\tan\left[\frac{1}{2}(e+fx)\right]^2] - 2(1+m) \operatorname{AppellF1}\left[\frac{1}{2}(3+m-n), -n, 2+m, \frac{1}{2}(5+m-n), \right. \\ & \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + (3+m-n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m-n), -n, \right. \\ & \left. 1+m, \frac{1}{2}(3+m-n), \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \cot\left[\frac{1}{2}(e+fx)\right]^2\right]\Big) \end{aligned}$$

**Problem 46:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (d \cot[e+fx])^n \sin[e+fx]^2 dx$$

Optimal (type 5, 51 leaves, 2 steps):

$$\frac{(d \cot[e+fx])^{1+n} \operatorname{Hypergeometric2F1}[2, \frac{1+n}{2}, \frac{3+n}{2}, -\cot[e+fx]^2]}{d f (1+n)}$$

Result (type 6, 5097 leaves):

$$\begin{aligned} & \left(8(-3+n) \cos\left[\frac{1}{2}(e+fx)\right]^5 (d \cot[e+fx])^n \sin\left[\frac{1}{2}(e+fx)\right] \right. \\ & \left( -\frac{1}{4} \cos[2(e+fx)]^3 \cot[e+fx]^n + \frac{1}{4} \cot[e+fx]^n \sin[2(e+fx)] + \right. \\ & \left. \frac{1}{2} \cot[e+fx]^n \sin[2(e+fx)]^2 - \frac{1}{4} \cot[e+fx]^n \sin[2(e+fx)]^3 + \right. \\ & \left. \cos[2(e+fx)]^2 \left(\frac{1}{2} \cot[e+fx]^n - \frac{1}{4} \cot[e+fx]^n \sin[2(e+fx)]\right) + \right. \\ & \left. \cos[2(e+fx)] \left(-\frac{1}{4} \cot[e+fx]^n - \frac{1}{4} \cot[e+fx]^n \sin[2(e+fx)]^2\right) \right) \\ & \left( - \left( \left( \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 2, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \right) / \right. \right. \\ & \left. \left( (-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 2, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ & \left. \left. 2 \left(n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-n, 2, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\ & \left. \left. \left. 2 \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, 3, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\ & \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2\right) + \right. \\ & \left. \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] / \right. \\ & \left. \left( (-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ & \left. \left. 2 \left(n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-n, 3, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\ & \left. \left. \left. 3 \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, 4, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \right) \end{aligned}$$



$$\begin{aligned}
& \left( - \left( \left( \text{AppellF1} \left[ \frac{1-n}{2}, -n, 2, \frac{3-n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \sec \left[ \frac{1}{2} (e+fx) \right]^2 \right) \Big/ \left( (-3+n) \text{AppellF1} \left[ \frac{1-n}{2}, -n, 2, \frac{3-n}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + 2 \left( n \text{AppellF1} \left[ \frac{3-n}{2}, 1-n, 2, \right. \right. \\
& \quad \left. \left. \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + 2 \text{AppellF1} \left[ \frac{3-n}{2}, -n, \right. \right. \\
& \quad \left. \left. 3, \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right) \tan \left[ \frac{1}{2} (e+fx) \right]^2 \right) + \\
& \text{AppellF1} \left[ \frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \Big/ \\
& \left( (-3+n) \text{AppellF1} \left[ \frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad \left. 2 \left( n \text{AppellF1} \left[ \frac{3-n}{2}, 1-n, 3, \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + 3 \right. \right. \\
& \quad \left. \left. \text{AppellF1} \left[ \frac{3-n}{2}, -n, 4, \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right) \tan \left[ \frac{1}{2} (e+fx) \right]^2 \right) + \\
& \frac{1}{-1+n} 8 (-3+n) \cos \left[ \frac{1}{2} (e+fx) \right]^5 \cot [e+fx]^n \sin \left[ \frac{1}{2} (e+fx) \right] \\
& \left( - \left( \left( \text{AppellF1} \left[ \frac{1-n}{2}, -n, 2, \frac{3-n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \sec \left[ \frac{1}{2} (e+fx) \right]^2 \tan \left[ \frac{1}{2} (e+fx) \right] \right) \Big/ \left( (-3+n) \text{AppellF1} \left[ \frac{1-n}{2}, -n, 2, \frac{3-n}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + 2 \left( n \text{AppellF1} \left[ \frac{3-n}{2}, 1-n, 2, \right. \right. \\
& \quad \left. \left. \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + 2 \text{AppellF1} \left[ \frac{3-n}{2}, -n, \right. \right. \\
& \quad \left. \left. 3, \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right) \tan \left[ \frac{1}{2} (e+fx) \right]^2 \right) - \\
& \left( \sec \left[ \frac{1}{2} (e+fx) \right]^2 \left( - \frac{1}{3-n} (1-n) n \text{AppellF1} \left[ 1 + \frac{1-n}{2}, 1-n, 2, 1 + \frac{3-n}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right) \sec \left[ \frac{1}{2} (e+fx) \right]^2 \tan \left[ \frac{1}{2} (e+fx) \right] - \right. \\
& \quad \left. \frac{1}{3-n} 2 (1-n) \text{AppellF1} \left[ 1 + \frac{1-n}{2}, -n, 3, 1 + \frac{3-n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right) \sec \left[ \frac{1}{2} (e+fx) \right]^2 \tan \left[ \frac{1}{2} (e+fx) \right] \right) \Big/ \\
& \left( (-3+n) \text{AppellF1} \left[ \frac{1-n}{2}, -n, 2, \frac{3-n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad \left. 2 \left( n \text{AppellF1} \left[ \frac{3-n}{2}, 1-n, 2, \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] + 2 \right. \right. \\
& \quad \left. \left. \text{AppellF1} \left[ \frac{3-n}{2}, -n, 3, \frac{5-n}{2}, \tan \left[ \frac{1}{2} (e+fx) \right]^2, -\tan \left[ \frac{1}{2} (e+fx) \right]^2 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \tan\left[\frac{1}{2}(e + fx)\right]^2\Big) + \left(-\frac{1}{3-n}(1-n)n \operatorname{AppellF1}\left[1 + \frac{1-n}{2}, 1-n, 3, 1 + \frac{3-n}{2}, \right.\right. \\
& \left.\left. \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \sec\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right]\right. \\
& \left.- \frac{1}{3-n} 3(1-n) \operatorname{AppellF1}\left[1 + \frac{1-n}{2}, -n, 4, 1 + \frac{3-n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, \right.\right. \\
& \left.\left. -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \sec\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right]\right) \Big/ \\
& \left((-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] + \right. \\
& \left. 2 \left(n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-n, 3, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] + 3 \right. \right. \\
& \left.\left. \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, 4, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, \right.\right. \right. \\
& \left.\left.\left. -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \tan\left[\frac{1}{2}(e + fx)\right]^2\right) + \right. \\
& \left(\operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 2, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \right. \\
& \left. \sec\left[\frac{1}{2}(e + fx)\right]^2 \left(2 \left(n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-n, 2, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, \right.\right. \right. \right. \\
& \left.\left.\left.\left. -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, 3, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, \right.\right. \right. \right. \\
& \left.\left.\left.\left. -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \right) \sec\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right] + \right. \\
& \left((-3+n) \left(-\frac{1}{3-n}(1-n)n \operatorname{AppellF1}\left[1 + \frac{1-n}{2}, 1-n, 2, 1 + \frac{3-n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, \right.\right. \right. \right. \\
& \left.\left.\left.\left. -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \sec\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right] - \frac{1}{3-n} 2(1-n) \right. \right. \right. \\
& \left.\left.\left. \operatorname{AppellF1}\left[1 + \frac{1-n}{2}, -n, 3, 1 + \frac{3-n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \right. \right. \right. \\
& \left. \sec\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right]\right) + 2 \tan\left[\frac{1}{2}(e + fx)\right]^2 \right. \\
& \left(\left(n \left(-\frac{1}{5-n} 2(3-n) \operatorname{AppellF1}\left[1 + \frac{3-n}{2}, 1-n, 3, 1 + \frac{5-n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, \right.\right. \right. \right. \right. \\
& \left.\left.\left.\left. \left. -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \sec\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right]\right) + \frac{1}{5-n} \right. \right. \right. \\
& \left.\left.\left. (1-n)(3-n) \operatorname{AppellF1}\left[1 + \frac{3-n}{2}, 2-n, 2, 1 + \frac{5-n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, \right.\right. \right. \right. \\
& \left.\left.\left. \left. -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \sec\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right]\right) + \right. \right. \right. \\
& \left. 2 \left(-\frac{1}{5-n}(3-n)n \operatorname{AppellF1}\left[1 + \frac{3-n}{2}, 1-n, 3, 1 + \frac{5-n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, \right.\right. \right. \right. \\
& \left.\left.\left. \left. -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \sec\left[\frac{1}{2}(e + fx)\right]^2 \tan\left[\frac{1}{2}(e + fx)\right]\right) - \frac{1}{5-n} \right. \right. \right. \\
& \left. 3(3-n) \operatorname{AppellF1}\left[1 + \frac{3-n}{2}, -n, 4, 1 + \frac{5-n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, \right.\right. \right. \right. 
\end{aligned}$$

$$\begin{aligned}
& -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)\right)\right)\Big) \\
& \left((-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 2, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& 2 \left(n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-n, 2, \frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& 2 \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, 3, \frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \left.\left.-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\Big)^2 - \\
& \left(\operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \left(2 \left(n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-n, 3, \frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \left.3 \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, 4, \frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \\
& \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + (-3+n) \\
& \left(-\frac{1}{3-n}(1-n) n \operatorname{AppellF1}\left[1+\frac{1-n}{2}, 1-n, 3, 1+\frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left.\left.-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3-n} 3 (1-n) \right. \\
& \operatorname{AppellF1}\left[1+\frac{1-n}{2}, -n, 4, 1+\frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \\
& \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\Big) + 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left(n \left(-\frac{1}{5-n} 3 (3-n) \operatorname{AppellF1}\left[1+\frac{3-n}{2}, 1-n, 4, 1+\frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left.\left.\left.-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5-n} \right. \right. \\
& \left.\left.(1-n) (3-n) \operatorname{AppellF1}\left[1+\frac{3-n}{2}, 2-n, 3, 1+\frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left.\left.\left.-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right) + \right. \\
& 3 \left(-\frac{1}{5-n} (3-n) n \operatorname{AppellF1}\left[1+\frac{3-n}{2}, 1-n, 4, 1+\frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left.\left.-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{5-n} \right. \\
& 4 (3-n) \operatorname{AppellF1}\left[1+\frac{3-n}{2}, -n, 5, 1+\frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \left.-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\Big)\Big)\Big)
\end{aligned}$$

$$3 \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, 4, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right)$$

**Problem 47:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (d \cot[e+fx])^n \sin[e+fx]^4 dx$$

Optimal (type 5, 51 leaves, 2 steps):

$$\frac{(d \cot[e+fx])^{1+n} \operatorname{Hypergeometric2F1}[3, \frac{1+n}{2}, \frac{3+n}{2}, -\cot[e+fx]^2]}{d f (1+n)}$$

Result (type 6, 8475 leaves):

$$\begin{aligned} & \left(2^{5-n} (-3+n) \cot[e+fx]^{-n} (d \cot[e+fx])^n \left(\cos[4(e+fx)] \right.\right. \\ & \left.\left. \left(\frac{1}{16} \cot[e+fx]^n - \frac{1}{4} i \cot[e+fx]^n \sin[2(e+fx)] - \frac{3}{8} \cot[e+fx]^n \sin[2(e+fx)]^2 + \right.\right. \\ & \left.\left. \frac{1}{4} i \cot[e+fx]^n \sin[2(e+fx)]^3 + \frac{1}{16} \cot[e+fx]^n \sin[2(e+fx)]^4\right) - \right. \\ & \left. \frac{1}{16} i \cot[e+fx]^n \sin[4(e+fx)] - \frac{1}{4} \cot[e+fx]^n \sin[2(e+fx)] \sin[4(e+fx)] + \right. \\ & \left. \frac{3}{8} i \cot[e+fx]^n \sin[2(e+fx)]^2 \sin[4(e+fx)] + \right. \\ & \left. \frac{1}{4} \cot[e+fx]^n \sin[2(e+fx)]^3 \sin[4(e+fx)] - \right. \\ & \left. \frac{1}{16} i \cot[e+fx]^n \sin[2(e+fx)]^4 \sin[4(e+fx)] + \right. \\ & \left. \cos[2(e+fx)]^4 \left(\frac{1}{16} \cos[4(e+fx)] \cot[e+fx]^n - \frac{1}{16} i \cot[e+fx]^n \sin[4(e+fx)]\right) + \right. \\ & \left. \cos[2(e+fx)]^3 \left(\cos[4(e+fx)] \left(-\frac{1}{4} \cot[e+fx]^n + \frac{1}{4} i \cot[e+fx]^n \sin[2(e+fx)]\right) + \right. \right. \\ & \left. \left. \frac{1}{4} i \cot[e+fx]^n \sin[4(e+fx)] + \frac{1}{4} \cot[e+fx]^n \sin[2(e+fx)] \sin[4(e+fx)]\right) + \right. \\ & \left. \cos[2(e+fx)]^2 \left(\cos[4(e+fx)] \left(\frac{3}{8} \cot[e+fx]^n - \frac{3}{4} i \cot[e+fx]^n \sin[2(e+fx)]\right) - \right. \right. \\ & \left. \left. \frac{3}{8} \cot[e+fx]^n \sin[2(e+fx)]^2\right) - \frac{3}{8} i \cot[e+fx]^n \sin[4(e+fx)] - \frac{3}{4} \cot[e+fx]^n \right. \\ & \left. \sin[2(e+fx)] \sin[4(e+fx)] + \frac{3}{8} i \cot[e+fx]^n \sin[2(e+fx)]^2 \sin[4(e+fx)]\right) + \\ & \left. \cos[2(e+fx)] \left(\cos[4(e+fx)] \left(-\frac{1}{4} \cot[e+fx]^n + \frac{3}{4} i \cot[e+fx]^n \sin[2(e+fx)]\right) + \right. \right. \\ & \left. \left. \frac{3}{4} \cot[e+fx]^n \sin[2(e+fx)]^2 - \frac{1}{4} i \cot[e+fx]^n \sin[2(e+fx)]^3\right) + \right. \end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} \operatorname{i} \operatorname{Cot}[e + f x]^n \sin[4(e + f x)] + \frac{3}{4} \operatorname{Cot}[e + f x]^n \sin[2(e + f x)] \sin[4(e + f x)] - \\
& \frac{3}{4} \operatorname{i} \operatorname{Cot}[e + f x]^n \sin[2(e + f x)]^2 \sin[4(e + f x)] - \\
& \frac{1}{4} \operatorname{Cot}[e + f x]^n \sin[2(e + f x)]^3 \sin[4(e + f x)] \Big) \\
& \left( \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right)^n \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \\
& \left( - \left( \left( \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \right. \right. \right. \\
& \quad \left. \left. \left. \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right)^2 \right) \right) / \\
& \quad \left( (-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + \right. \\
& \quad 2 \left( n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-n, 3, \frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + \right. \\
& \quad \left. \left. \left. 3 \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, 4, \frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \right) \right. \\
& \quad \left( \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \Big) + \left( 2 \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 4, \frac{3-n}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \right) / \\
& \quad \left( (-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 4, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + \right. \\
& \quad 2 \left( n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-n, 4, \frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + 4 \operatorname{AppellF1}\left[\right. \right. \\
& \quad \left. \left. \frac{3-n}{2}, -n, 5, \frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) - \\
& \quad \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 5, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] / \\
& \quad \left( (-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 5, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + \right. \\
& \quad 2 \left( n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-n, 5, \frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + \right. \\
& \quad \left. \left. 5 \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, 6, \frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \right) \\
& \quad \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \Big) \Big) / \left( f (-1+n) \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right)^5 \right. \\
& \quad \left( - \frac{1}{(-1+n) \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right)^6} 5 \times 2^{5-n} (-3+n) \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right. \\
& \quad \left. \left( \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right)^n \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right. \\
& \quad \left. \left( - \left( \left( \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \right) \right) \right) \right)
\end{aligned}$$



$$\begin{aligned}
& \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2] + 4 \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, \right. \\
& \left. 5, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \tan\left[\frac{1}{2}(e+f x)\right]^2] - \\
& \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 5, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] / \\
& \left((-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 5, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \\
& \left. 2\left(n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-n, 5, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + 5 \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, 6, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right]\right) \right. \\
& \left. \tan\left[\frac{1}{2}(e+f x)\right]^2\right) + \frac{1}{(-1+n)\left(1+\tan\left[\frac{1}{2}(e+f x)\right]^2\right)^5} \\
& 2^{5-n} (-3+n) n \left(-\frac{1}{2} \csc\left[\frac{1}{2}(e+f x)\right]^2 - \frac{1}{2} \sec\left[\frac{1}{2}(e+f x)\right]^2\right) \\
& \left(\cot\left[\frac{1}{2}(e+f x)\right] - \tan\left[\frac{1}{2}(e+f x)\right]\right)^{-1+n} \tan\left[\frac{1}{2}(e+f x)\right] \\
& \left(-\left(\left(\operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \right. \right. \right. \\
& \left. \left. \left. \left(1+\tan\left[\frac{1}{2}(e+f x)\right]^2\right)^2\right) / \left((-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \right. \right. \right. \\
& \left. \left. \left.\tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + 2\left(n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-n, 3, \right. \right. \right. \\
& \left. \left. \left.\frac{5-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + 3 \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, \right. \right. \right. \\
& \left. \left. \left. 4, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right)\right) \tan\left[\frac{1}{2}(e+f x)\right]^2\right) + \\
& \left(2 \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 4, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \right. \\
& \left.\left(1+\tan\left[\frac{1}{2}(e+f x)\right]^2\right)\right) / \left((-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 4, \frac{3-n}{2}, \right. \right. \\
& \left. \left.\tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + 2\left(n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-n, 4, \right. \right. \right. \\
& \left. \left. \left.\frac{5-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + 4 \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, \right. \right. \right. \\
& \left. \left. \left. 5, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right)\right) \tan\left[\frac{1}{2}(e+f x)\right]^2\right) - \\
& \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 5, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] / \\
& \left((-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -n, 5, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \\
& \left. 2\left(n \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-n, 5, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] + 5 \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3-n}{2}, -n, 6, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, \right.\right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{(-1+n) \left(1+\tan\left(\frac{1}{2} (e+f x)\right)^2\right)^5} 2^{5-n} (-3+n) \left(\cot\left(\frac{1}{2} (e+f x)\right) - \tan\left(\frac{1}{2} (e+f x)\right)\right)^n \\
& \tan\left(\frac{1}{2} (e+f x)\right) \\
& \left(-\left(\left(2 \text{AppellF1}\left[\frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \tan\left(\frac{1}{2} (e+f x)\right)^2, -\tan\left(\frac{1}{2} (e+f x)\right)^2\right]\right.\right. \right. \\
& \sec\left(\frac{1}{2} (e+f x)\right)^2 \tan\left(\frac{1}{2} (e+f x)\right) \left(1+\tan\left(\frac{1}{2} (e+f x)\right)^2\right)\Big)\Big) / \\
& \left((-3+n) \text{AppellF1}\left[\frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \tan\left(\frac{1}{2} (e+f x)\right)^2, -\tan\left(\frac{1}{2} (e+f x)\right)^2\right] + \right. \\
& 2 \left(n \text{AppellF1}\left[\frac{3-n}{2}, 1-n, 3, \frac{5-n}{2}, \tan\left(\frac{1}{2} (e+f x)\right)^2, -\tan\left(\frac{1}{2} (e+f x)\right)^2\right] + \right. \\
& 3 \text{AppellF1}\left[\frac{3-n}{2}, -n, 4, \frac{5-n}{2}, \tan\left(\frac{1}{2} (e+f x)\right)^2, \right. \\
& \left.-\tan\left(\frac{1}{2} (e+f x)\right)^2\right] \tan\left(\frac{1}{2} (e+f x)\right)^2\Big) - \\
& \left(\left(-\frac{1}{3-n} (1-n) n \text{AppellF1}\left[1+\frac{1-n}{2}, 1-n, 3, 1+\frac{3-n}{2}, \tan\left(\frac{1}{2} (e+f x)\right)^2, \right.\right. \right. \\
& \left.-\tan\left(\frac{1}{2} (e+f x)\right)^2\right] \sec\left(\frac{1}{2} (e+f x)\right)^2 \tan\left(\frac{1}{2} (e+f x)\right) - \frac{1}{3-n} 3 (1-n) \right. \\
& \text{AppellF1}\left[1+\frac{1-n}{2}, -n, 4, 1+\frac{3-n}{2}, \tan\left(\frac{1}{2} (e+f x)\right)^2, -\tan\left(\frac{1}{2} (e+f x)\right)^2\right] \\
& \sec\left(\frac{1}{2} (e+f x)\right)^2 \tan\left(\frac{1}{2} (e+f x)\right) \left(1+\tan\left(\frac{1}{2} (e+f x)\right)^2\right)^2\Big) / \\
& \left((-3+n) \text{AppellF1}\left[\frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \tan\left(\frac{1}{2} (e+f x)\right)^2, -\tan\left(\frac{1}{2} (e+f x)\right)^2\right] + \right. \\
& 2 \left(n \text{AppellF1}\left[\frac{3-n}{2}, 1-n, 3, \frac{5-n}{2}, \tan\left(\frac{1}{2} (e+f x)\right)^2, -\tan\left(\frac{1}{2} (e+f x)\right)^2\right] + 3 \right. \\
& \text{AppellF1}\left[\frac{3-n}{2}, -n, 4, \frac{5-n}{2}, \tan\left(\frac{1}{2} (e+f x)\right)^2, -\tan\left(\frac{1}{2} (e+f x)\right)^2\Big] \Big) \\
& \tan\left(\frac{1}{2} (e+f x)\right)^2 + \left(2 \text{AppellF1}\left[\frac{1-n}{2}, -n, 4, \frac{3-n}{2}, \tan\left(\frac{1}{2} (e+f x)\right)^2, \right.\right. \\
& \left.-\tan\left(\frac{1}{2} (e+f x)\right)^2\right] \sec\left(\frac{1}{2} (e+f x)\right)^2 \tan\left(\frac{1}{2} (e+f x)\right)\Big) / \\
& \left((-3+n) \text{AppellF1}\left[\frac{1-n}{2}, -n, 4, \frac{3-n}{2}, \tan\left(\frac{1}{2} (e+f x)\right)^2, -\tan\left(\frac{1}{2} (e+f x)\right)^2\right] + \right. \\
& 2 \left(n \text{AppellF1}\left[\frac{3-n}{2}, 1-n, 4, \frac{5-n}{2}, \tan\left(\frac{1}{2} (e+f x)\right)^2, -\tan\left(\frac{1}{2} (e+f x)\right)^2\right] + 4 \right. \\
& \text{AppellF1}\left[\frac{3-n}{2}, -n, 5, \frac{5-n}{2}, \tan\left(\frac{1}{2} (e+f x)\right)^2, \right. \\
& \left.-\tan\left(\frac{1}{2} (e+f x)\right)^2\right] \tan\left(\frac{1}{2} (e+f x)\right)^2 + \\
& \left(2 \left(-\frac{1}{3-n} (1-n) n \text{AppellF1}\left[1+\frac{1-n}{2}, 1-n, 4, 1+\frac{3-n}{2}, \tan\left(\frac{1}{2} (e+f x)\right)^2, \right.\right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3-n} 4(1-n) \\
& \text{AppellF1}\left[1 + \frac{1-n}{2}, -n, 5, 1 + \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \\
& \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \Big) \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right) \Big) / \\
& \left((-3+n) \text{AppellF1}\left[\frac{1-n}{2}, -n, 4, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& 2 \left(n \text{AppellF1}\left[\frac{3-n}{2}, 1-n, 4, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 4 \right. \\
& \left.\text{AppellF1}\left[\frac{3-n}{2}, -n, 5, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \\
& \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big) - \left(-\frac{1}{3-n}(1-n) n \text{AppellF1}\left[1 + \frac{1-n}{2}, 1-n, 5, 1 + \frac{3-n}{2}, \right. \right. \\
& \left.\tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \\
& \frac{1}{3-n} 5(1-n) \text{AppellF1}\left[1 + \frac{1-n}{2}, -n, 6, 1 + \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \left.-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \Big) / \\
& \left((-3+n) \text{AppellF1}\left[\frac{1-n}{2}, -n, 5, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& 2 \left(n \text{AppellF1}\left[\frac{3-n}{2}, 1-n, 5, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 5 \right. \\
& \left.\text{AppellF1}\left[\frac{3-n}{2}, -n, 6, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left.-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big) + \\
& \left(\text{AppellF1}\left[\frac{1-n}{2}, -n, 3, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \left(1 + \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \left(2 \left(n \text{AppellF1}\left[\frac{3-n}{2}, 1-n, 3, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left.-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 3 \text{AppellF1}\left[\frac{3-n}{2}, -n, 4, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \left.-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& (-3+n) \left(-\frac{1}{3-n}(1-n) n \text{AppellF1}\left[1 + \frac{1-n}{2}, 1-n, 3, 1 + \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left.-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3-n} 3(1-n) \\
& \text{AppellF1}\left[1 + \frac{1-n}{2}, -n, 4, 1 + \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \\
& \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \Big) + 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left(n \left(-\frac{1}{5-n} 3(3-n) \text{AppellF1}\left[1 + \frac{3-n}{2}, 1-n, 4, 1 + \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right.
\end{aligned}$$





$$\begin{aligned} & -\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]\right)\right)\right)\Big) \\ & \left((-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2},-n,5,\frac{3-n}{2},\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2,-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2\right]+2\left(n \operatorname{AppellF1}\left[\frac{3-n}{2},1-n,5,\frac{5-n}{2},\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2,-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2\right]+5 \operatorname{AppellF1}\left[\frac{3-n}{2},-n,6,\frac{5-n}{2},\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2,-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2\right)\Big)\Big) \end{aligned}$$

**Problem 48: Result more than twice size of optimal antiderivative.**

$$\int (\mathbf{d} \operatorname{Cot}[\mathbf{e}+\mathbf{f} \mathbf{x}])^n \operatorname{Csc}[\mathbf{e}+\mathbf{f} \mathbf{x}]^3 d\mathbf{x}$$

Optimal (type 5, 79 leaves, 1 step):

$$\begin{aligned} & -\frac{1}{\mathbf{d} \mathbf{f} (1+n)} (\mathbf{d} \operatorname{Cot}[\mathbf{e}+\mathbf{f} \mathbf{x}])^{1+n} \operatorname{Csc}[\mathbf{e}+\mathbf{f} \mathbf{x}]^3 \\ & \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, \frac{4+n}{2}, \frac{3+n}{2}, \operatorname{Cos}[\mathbf{e}+\mathbf{f} \mathbf{x}]^2\right] (\operatorname{Sin}[\mathbf{e}+\mathbf{f} \mathbf{x}]^2)^{\frac{4+n}{2}} \end{aligned}$$

Result (type 5, 190 leaves):

$$\begin{aligned} & -\frac{1}{4 \mathbf{f} n (-4+n^2)} (\mathbf{d} \operatorname{Cot}[\mathbf{e}+\mathbf{f} \mathbf{x}])^n \\ & \left((-2+n) n \operatorname{Cot}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^4 \operatorname{Hypergeometric2F1}\left[-1-\frac{n}{2},-n,-\frac{n}{2},\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2\right]+\right. \\ & (2+n) \left(n \operatorname{Hypergeometric2F1}\left[1-\frac{n}{2},-n,2-\frac{n}{2},\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2\right]+\right. \\ & 2(-2+n) \operatorname{Cot}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2 \operatorname{Hypergeometric2F1}\left[-n,-\frac{n}{2},1-\frac{n}{2},\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2\right]\Big) \\ & \left(\operatorname{Cos}[\mathbf{e}+\mathbf{f} \mathbf{x}] \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2\right)^{-n} \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} \mathbf{x})\right]^2 \end{aligned}$$

**Problem 50: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (\mathbf{d} \operatorname{Cot}[\mathbf{e}+\mathbf{f} \mathbf{x}])^n \operatorname{Sin}[\mathbf{e}+\mathbf{f} \mathbf{x}] d\mathbf{x}$$

Optimal (type 5, 73 leaves, 1 step):

$$\begin{aligned} & -\frac{1}{\mathbf{d} \mathbf{f} (1+n)} (\mathbf{d} \operatorname{Cot}[\mathbf{e}+\mathbf{f} \mathbf{x}])^{1+n} \\ & \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \operatorname{Cos}[\mathbf{e}+\mathbf{f} \mathbf{x}]^2\right] \operatorname{Sin}[\mathbf{e}+\mathbf{f} \mathbf{x}] (\operatorname{Sin}[\mathbf{e}+\mathbf{f} \mathbf{x}]^2)^{n/2} \end{aligned}$$

Result (type 6, 1973 leaves) :

$$\begin{aligned}
& \left. \left( -\operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \right) \operatorname{Sec}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right] \right) \Bigg) \Bigg) \Bigg) \\
& \left( (-2+n) \left( 2n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 2, 3-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \right] + \right. \right. \\
& \quad 4 \operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 3, 3-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \right] + \\
& \quad (-4+n) \operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 2, 2-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2, \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \right] \operatorname{Cot}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \right) \Bigg) + \\
& \left. \left( 4(-4+n) \operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 2, 2-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \right] \right. \right. \\
& \quad \cos\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^4 \operatorname{Cot}[\operatorname{e} + \operatorname{f} x]^n \left( -(-4+n) \operatorname{AppellF1}\left[1-\frac{n}{2}, -n, 2, 2-\frac{n}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \right] \operatorname{Cot}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right] \operatorname{Csc}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 + \right. \\
& \quad (-4+n) \operatorname{Cot}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \left( -\frac{1}{2-\frac{n}{2}} \left(1-\frac{n}{2}\right) n \operatorname{AppellF1}\left[2-\frac{n}{2}, 1-n, 2, 3-\frac{n}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right] - \right. \\
& \quad \left. \left. \frac{1}{2-\frac{n}{2}} 2 \left(1-\frac{n}{2}\right) \operatorname{AppellF1}\left[2-\frac{n}{2}, -n, 3, 3-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right] \right) + \right. \\
& \quad \left. \left. 2n \left( -\frac{1}{3-\frac{n}{2}} 2 \left(2-\frac{n}{2}\right) \operatorname{AppellF1}\left[3-\frac{n}{2}, 1-n, 3, 4-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right] + \frac{1}{3-\frac{n}{2}} (1-n) \left(2-\frac{n}{2}\right) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1}\left[3-\frac{n}{2}, 2-n, 2, 4-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \right] \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Sec}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right] \right) + 4 \left( -\frac{1}{3-\frac{n}{2}} \left(2-\frac{n}{2}\right) n \operatorname{AppellF1}\left[3-\frac{n}{2}, 1-n, \right. \right. \right. \\
& \quad \left. \left. \left. 3, 4-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right] - \right. \right. \\
& \quad \left. \left. \left. \frac{1}{3-\frac{n}{2}} 3 \left(2-\frac{n}{2}\right) \operatorname{AppellF1}\left[3-\frac{n}{2}, -n, 4, 4-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\operatorname{e} + \operatorname{f} x)\right] \right) \right) \right) \Bigg)
\end{aligned}$$

$$\left( \left( -2 + n \right) \left( 2 n \operatorname{AppellF1} \left[ 2 - \frac{n}{2}, 1 - n, 2, 3 - \frac{n}{2}, \tan \left[ \frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[ \frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] + 4 \operatorname{AppellF1} \left[ 2 - \frac{n}{2}, -n, 3, 3 - \frac{n}{2}, \tan \left[ \frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[ \frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] + (-4 + n) \operatorname{AppellF1} \left[ 1 - \frac{n}{2}, -n, 2, 2 - \frac{n}{2}, \tan \left[ \frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2, -\tan \left[ \frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right] \right) \right)$$

**Problem 51:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (\mathbf{d} \cot[\mathbf{e} + \mathbf{f} x])^n \sin[\mathbf{e} + \mathbf{f} x]^3 dx$$

Optimal (type 5, 79 leaves, 1 step):

$$-\frac{1}{\mathbf{d} \mathbf{f} (1+n)} (\mathbf{d} \cot[\mathbf{e} + \mathbf{f} x])^{1+n}$$

$$\text{Hypergeometric2F1} \left[ \frac{1}{2} (-2+n), \frac{1+n}{2}, \frac{3+n}{2}, \cos[\mathbf{e} + \mathbf{f} x]^2 \right] \sin[\mathbf{e} + \mathbf{f} x]^3 (\sin[\mathbf{e} + \mathbf{f} x]^2)^{\frac{1}{2}(-2+n)}$$

Result (type 6, 5173 leaves):

$$\begin{aligned} & \left( 16 (-4+n) \cos \left[ \frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^6 (\mathbf{d} \cot[\mathbf{e} + \mathbf{f} x])^n \sin \left[ \frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \right. \\ & \left( \cos[3(\mathbf{e} + \mathbf{f} x)] \left( -\frac{1}{8} \mathbf{i} \cot[\mathbf{e} + \mathbf{f} x]^n - \frac{3}{8} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[2(\mathbf{e} + \mathbf{f} x)] \right. \right. \\ & \left. \left. + \frac{3}{8} \mathbf{i} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[2(\mathbf{e} + \mathbf{f} x)]^2 + \frac{1}{8} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[2(\mathbf{e} + \mathbf{f} x)]^3 \right) - \right. \\ & \left. \frac{1}{8} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[3(\mathbf{e} + \mathbf{f} x)] + \frac{3}{8} \mathbf{i} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[2(\mathbf{e} + \mathbf{f} x)] \sin[3(\mathbf{e} + \mathbf{f} x)] + \right. \\ & \left. \frac{3}{8} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[2(\mathbf{e} + \mathbf{f} x)]^2 \sin[3(\mathbf{e} + \mathbf{f} x)] - \right. \\ & \left. \frac{1}{8} \mathbf{i} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[2(\mathbf{e} + \mathbf{f} x)]^3 \sin[3(\mathbf{e} + \mathbf{f} x)] + \right. \\ & \left. \cos[2(\mathbf{e} + \mathbf{f} x)]^3 \left( \frac{1}{8} \mathbf{i} \cos[3(\mathbf{e} + \mathbf{f} x)] \cot[\mathbf{e} + \mathbf{f} x]^n + \frac{1}{8} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[3(\mathbf{e} + \mathbf{f} x)] \right) + \right. \\ & \left. \cos[2(\mathbf{e} + \mathbf{f} x)]^2 \left( \cos[3(\mathbf{e} + \mathbf{f} x)] \left( -\frac{3}{8} \mathbf{i} \cot[\mathbf{e} + \mathbf{f} x]^n - \frac{3}{8} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[2(\mathbf{e} + \mathbf{f} x)] \right) - \right. \right. \\ & \left. \left. \frac{3}{8} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[3(\mathbf{e} + \mathbf{f} x)] + \frac{3}{8} \mathbf{i} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[2(\mathbf{e} + \mathbf{f} x)] \sin[3(\mathbf{e} + \mathbf{f} x)] \right) + \right. \\ & \left. \cos[2(\mathbf{e} + \mathbf{f} x)] \left( \cos[3(\mathbf{e} + \mathbf{f} x)] \left( \frac{3}{8} \mathbf{i} \cot[\mathbf{e} + \mathbf{f} x]^n + \frac{3}{4} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[2(\mathbf{e} + \mathbf{f} x)] - \frac{3}{8} \right. \right. \right. \\ & \left. \left. \left. \mathbf{i} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[2(\mathbf{e} + \mathbf{f} x)]^2 + \frac{3}{8} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[3(\mathbf{e} + \mathbf{f} x)] - \frac{3}{4} \mathbf{i} \cot[\mathbf{e} + \mathbf{f} x]^n \right. \right. \right. \\ & \left. \left. \left. \sin[2(\mathbf{e} + \mathbf{f} x)] \sin[3(\mathbf{e} + \mathbf{f} x)] - \frac{3}{8} \cot[\mathbf{e} + \mathbf{f} x]^n \sin[2(\mathbf{e} + \mathbf{f} x)]^2 \sin[3(\mathbf{e} + \mathbf{f} x)] \right) \right) \right) \end{aligned}$$

$$\begin{aligned}
& - \left( \left( \text{AppellF1} \left[ 1 - \frac{n}{2}, -n, 3, 2 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] \sec \left[ \frac{1}{2} (e + fx) \right]^2 \right) \right. \\
& \quad \left( (-4+n) \text{AppellF1} \left[ 1 - \frac{n}{2}, -n, 3, 2 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] + \right. \\
& \quad \left. 2 \left( n \text{AppellF1} \left[ 2 - \frac{n}{2}, 1 - n, 3, 3 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 3 \text{AppellF1} \left[ 2 - \frac{n}{2}, -n, 4, 3 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (e + fx) \right]^2 \right) + \\
& \text{AppellF1} \left[ 1 - \frac{n}{2}, -n, 4, 2 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] / \\
& \left( (-4+n) \text{AppellF1} \left[ 1 - \frac{n}{2}, -n, 4, 2 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] + \right. \\
& \quad \left. 2 \left( n \text{AppellF1} \left[ 2 - \frac{n}{2}, 1 - n, 4, 3 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 4 \text{AppellF1} \left[ 2 - \frac{n}{2}, -n, 5, 3 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (e + fx) \right]^2 \right) \right) / \\
& \left( f (-2+n) \left( \frac{1}{-2+n} 16 (-4+n) \cos \left[ \frac{1}{2} (e + fx) \right]^7 \cot [e + fx]^n \sin \left[ \frac{1}{2} (e + fx) \right] \right. \right. \\
& \quad \left( \left( \text{AppellF1} \left[ 1 - \frac{n}{2}, -n, 3, 2 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \sec \left[ \frac{1}{2} (e + fx) \right]^2 \right) / \left( (-4+n) \text{AppellF1} \left[ 1 - \frac{n}{2}, -n, 3, 2 - \frac{n}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] + 2 \left( n \text{AppellF1} \left[ 2 - \frac{n}{2}, 1 - n, 3, \right. \right. \\
& \quad \left. \left. 3 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] + 3 \text{AppellF1} \left[ 2 - \frac{n}{2}, -n, \right. \right. \\
& \quad \left. \left. 4, 3 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (e + fx) \right]^2 \right) \right) + \\
& \text{AppellF1} \left[ 1 - \frac{n}{2}, -n, 4, 2 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] / \\
& \left( (-4+n) \text{AppellF1} \left[ 1 - \frac{n}{2}, -n, 4, 2 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] + \right. \\
& \quad \left. 2 \left( n \text{AppellF1} \left[ 2 - \frac{n}{2}, 1 - n, 4, 3 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] + 4 \right. \right. \\
& \quad \left. \left. \text{AppellF1} \left[ 2 - \frac{n}{2}, -n, 5, 3 - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + fx) \right]^2, -\tan \left[ \frac{1}{2} (e + fx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (e + fx) \right]^2 \right) - \\
& \frac{1}{-2+n} 16 (-4+n) n \cos \left[ \frac{1}{2} (e + fx) \right]^6 \cot [e + fx]^{-1+n} \csc [e + fx]^2 \sin \left[ \frac{1}{2} (e + fx) \right]^2
\end{aligned}$$





$$\begin{aligned}
& \left. \left( \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& (-4+n) \left( -\frac{1}{2-\frac{n}{2}} \left( 1 - \frac{n}{2} \right) n \operatorname{AppellF1}\left[2 - \frac{n}{2}, 1 - n, 3, 3 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{2-\frac{n}{2}} 3 \left( 1 - \frac{n}{2} \right) \right. \\
& \operatorname{AppellF1}\left[2 - \frac{n}{2}, -n, 4, 3 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \\
& \left. \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left. \left( n \left( -\frac{1}{3 - \frac{n}{2}} 3 \left( 2 - \frac{n}{2} \right) \operatorname{AppellF1}\left[3 - \frac{n}{2}, 1 - n, 4, 4 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3 - \frac{n}{2}} \right. \right. \\
& \left. \left. \left( 1 - n \right) \left( 2 - \frac{n}{2} \right) \operatorname{AppellF1}\left[3 - \frac{n}{2}, 2 - n, 3, 4 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) + \right. \\
& \left. \left. \left. 3 \left( -\frac{1}{3 - \frac{n}{2}} \left( 2 - \frac{n}{2} \right) n \operatorname{AppellF1}\left[3 - \frac{n}{2}, 1 - n, 4, 4 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{3 - \frac{n}{2}} \right. \right. \right. \\
& \left. \left. \left. 4 \left( 2 - \frac{n}{2} \right) \operatorname{AppellF1}\left[3 - \frac{n}{2}, -n, 5, 4 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \right) \Big/ \\
& \left( (-4+n) \operatorname{AppellF1}\left[1 - \frac{n}{2}, -n, 3, 2 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] + \right. \\
& \left. 2 \left( n \operatorname{AppellF1}\left[2 - \frac{n}{2}, 1 - n, 3, 3 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] + \right. \right. \\
& \left. \left. 3 \operatorname{AppellF1}\left[2 - \frac{n}{2}, -n, 4, 3 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 - \right. \\
& \left. \left( \operatorname{AppellF1}\left[1 - \frac{n}{2}, -n, 4, 2 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right. \right. \\
& \left. \left. \left( 2 \left( n \operatorname{AppellF1}\left[2 - \frac{n}{2}, 1 - n, 4, 3 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] + \right. \right. \right. \right. \\
& \left. \left. \left. \left. 3 \operatorname{AppellF1}\left[2 - \frac{n}{2}, -n, 5, 4 - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \right) \right) \right)
\end{aligned}$$

Problem 52: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b \cot[e + f x])^n (a \csc[e + f x])^m dx$$

Optimal (type 5, 83 leaves, 1 step):

$$-\frac{1}{b f (1+n)} (b \cot[e + f x])^{1+n} (a \csc[e + f x])^m \\ \text{Hypergeometric2F1}\left[\frac{1+n}{2}, \frac{1}{2} (1+m+n), \frac{3+n}{2}, \cos[e + f x]^2\right] (\sin[e + f x]^2)^{\frac{1}{2}(1+m+n)}$$

Result (type 6, 3166 leaves):

$$-\left( \left( 2 (-3+m+n) \operatorname{AppellF1}\left[\frac{1}{2} (1-m-n), -n, 1-m, \frac{1}{2} (3-m-n), \tan[\frac{1}{2} (e+f x)]^2, \cot[\frac{1}{2} (e+f x)] \cot[e+f x]^n (b \cot[e+f x])^n \csc[e+f x]^m (a \csc[e+f x])^m \right] \right) / \right. \\ \left. \left( f (-1+m+n) \left( 2 n \operatorname{AppellF1}\left[\frac{1}{2} (3-m-n), 1-n, 1-m, \frac{1}{2} (5-m-n), \tan[\frac{1}{2} (e+f x)]^2, -\tan[\frac{1}{2} (e+f x)]^2 \right] - 2 (-1+m) \operatorname{AppellF1}\left[\frac{1}{2} (3-m-n), -n, 2-m, \frac{1}{2} (5-m-n), \tan[\frac{1}{2} (e+f x)]^2, -\tan[\frac{1}{2} (e+f x)]^2 \right] + (-3+m+n) \operatorname{AppellF1}\left[\frac{1}{2} (1-m-n), -n, 1-m, \frac{1}{2} (3-m-n), \tan[\frac{1}{2} (e+f x)]^2, -\tan[\frac{1}{2} (e+f x)]^2 \right] \cot[\frac{1}{2} (e+f x)]^2 \right) \right. \\ \left. \left( 2 (-3+m+n) \operatorname{AppellF1}\left[\frac{1}{2} (1-m-n), -n, 1-m, \frac{1}{2} (3-m-n), \tan[\frac{1}{2} (e+f x)]^2, -\tan[\frac{1}{2} (e+f x)]^2 \right] \cos[\frac{1}{2} (e+f x)]^2 \cot[e+f x]^n \csc[e+f x]^m \right) / \right. \\ \left. \left( (-1+m+n) \left( 2 n \operatorname{AppellF1}\left[\frac{1}{2} (3-m-n), 1-n, 1-m, \frac{1}{2} (5-m-n), \tan[\frac{1}{2} (e+f x)]^2, -\tan[\frac{1}{2} (e+f x)]^2 \right] - 2 (-1+m) \operatorname{AppellF1}\left[\frac{1}{2} (3-m-n), -n, 2-m, \frac{1}{2} (5-m-n), \tan[\frac{1}{2} (e+f x)]^2, -\tan[\frac{1}{2} (e+f x)]^2 \right] + (-3+m+n) \operatorname{AppellF1}\left[\frac{1}{2} (1-m-n), -n, 1-m, \frac{1}{2} (3-m-n), \tan[\frac{1}{2} (e+f x)]^2, -\tan[\frac{1}{2} (e+f x)]^2 \right] \cot[\frac{1}{2} (e+f x)]^2 \right) \right. \\ \left. \left( (-3+m+n) \operatorname{AppellF1}\left[\frac{1}{2} (1-m-n), -n, 1-m, \frac{1}{2} (3-m-n), \tan[\frac{1}{2} (e+f x)]^2, -\tan[\frac{1}{2} (e+f x)]^2 \right] \cot[\frac{1}{2} (e+f x)]^2 \cot[e+f x]^n \csc[e+f x]^m \right) / \right. \\ \left. \left( (-1+m+n) \left( 2 n \operatorname{AppellF1}\left[\frac{1}{2} (3-m-n), 1-n, 1-m, \frac{1}{2} (5-m-n), \tan[\frac{1}{2} (e+f x)]^2, -\tan[\frac{1}{2} (e+f x)]^2 \right] - 2 (-1+m) \operatorname{AppellF1}\left[\frac{1}{2} (3-m-n), -n, 2-m, \frac{1}{2} (5-m-n), \tan[\frac{1}{2} (e+f x)]^2, -\tan[\frac{1}{2} (e+f x)]^2 \right] + (-3+m+n) \operatorname{AppellF1}\left[\frac{1}{2} (1-m-n), -n, 1-m, \frac{1}{2} (3-m-n), \tan[\frac{1}{2} (e+f x)]^2, -\tan[\frac{1}{2} (e+f x)]^2 \right] \cot[\frac{1}{2} (e+f x)]^2 \right) \right)$$

$$\begin{aligned}
& \left. \left( 1 - m, \frac{1}{2} (3 - m - n), \tan\left[\frac{1}{2} (\epsilon + f x)\right]^2, -\tan\left[\frac{1}{2} (\epsilon + f x)\right]^2 \cot\left[\frac{1}{2} (\epsilon + f x)\right]^2 \right) + \right. \\
& \left( 2 m (-3 + m + n) \operatorname{AppellF1}\left[\frac{1}{2} (1 - m - n), -n, 1 - m, \frac{1}{2} (3 - m - n)\right], \right. \\
& \quad \left. \tan\left[\frac{1}{2} (\epsilon + f x)\right]^2, -\tan\left[\frac{1}{2} (\epsilon + f x)\right]^2 \cos\left[\frac{1}{2} (\epsilon + f x)\right]^2 \right. \\
& \quad \left. \cos[\epsilon + f x] \cot\left[\frac{1}{2} (\epsilon + f x)\right] \cot[\epsilon + f x]^n \csc[\epsilon + f x]^{1+m} \right) / \\
& \left( (-1 + m + n) \left( 2 n \operatorname{AppellF1}\left[\frac{1}{2} (3 - m - n), 1 - n, 1 - m, \frac{1}{2} (5 - m - n), \tan\left[\frac{1}{2} (\epsilon + f x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (\epsilon + f x)\right]^2 \right] - 2 (-1 + m) \operatorname{AppellF1}\left[\frac{1}{2} (3 - m - n), -n, 2 - m, \frac{1}{2} (5 - m - n), \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (\epsilon + f x)\right]^2, -\tan\left[\frac{1}{2} (\epsilon + f x)\right]^2 \right] + (-3 + m + n) \operatorname{AppellF1}\left[\frac{1}{2} (1 - m - n), -n, \right. \right. \\
& \quad \left. \left. 1 - m, \frac{1}{2} (3 - m - n), \tan\left[\frac{1}{2} (\epsilon + f x)\right]^2, -\tan\left[\frac{1}{2} (\epsilon + f x)\right]^2 \cot\left[\frac{1}{2} (\epsilon + f x)\right]^2 \right) \right) + \right. \\
& \left( 2 n (-3 + m + n) \operatorname{AppellF1}\left[\frac{1}{2} (1 - m - n), -n, 1 - m, \frac{1}{2} (3 - m - n), \right. \right. \\
& \quad \left. \tan\left[\frac{1}{2} (\epsilon + f x)\right]^2, -\tan\left[\frac{1}{2} (\epsilon + f x)\right]^2 \right. \\
& \quad \left. \cos\left[\frac{1}{2} (\epsilon + f x)\right]^2 \cot\left[\frac{1}{2} (\epsilon + f x)\right] \cot[\epsilon + f x]^{-1+n} \csc[\epsilon + f x]^{2+m} \right) / \\
& \left( (-1 + m + n) \left( 2 n \operatorname{AppellF1}\left[\frac{1}{2} (3 - m - n), 1 - n, 1 - m, \frac{1}{2} (5 - m - n), \tan\left[\frac{1}{2} (\epsilon + f x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (\epsilon + f x)\right]^2 \right] - 2 (-1 + m) \operatorname{AppellF1}\left[\frac{1}{2} (3 - m - n), -n, 2 - m, \frac{1}{2} (5 - m - n), \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (\epsilon + f x)\right]^2, -\tan\left[\frac{1}{2} (\epsilon + f x)\right]^2 \right] + (-3 + m + n) \operatorname{AppellF1}\left[\frac{1}{2} (1 - m - n), -n, \right. \right. \\
& \quad \left. \left. 1 - m, \frac{1}{2} (3 - m - n), \tan\left[\frac{1}{2} (\epsilon + f x)\right]^2, -\tan\left[\frac{1}{2} (\epsilon + f x)\right]^2 \cot\left[\frac{1}{2} (\epsilon + f x)\right]^2 \right) \right) - \right. \\
& \left( 2 (-3 + m + n) \cos\left[\frac{1}{2} (\epsilon + f x)\right]^2 \cot\left[\frac{1}{2} (\epsilon + f x)\right] \cot[\epsilon + f x]^n \csc[\epsilon + f x]^m \right. \\
& \quad \left( -\frac{1}{3 - m - n} (1 - m - n) n \operatorname{AppellF1}\left[1 + \frac{1}{2} (1 - m - n), 1 - n, 1 - m, 1 + \frac{1}{2} (3 - m - n), \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (\epsilon + f x)\right]^2, -\tan\left[\frac{1}{2} (\epsilon + f x)\right]^2 \right] \sec\left[\frac{1}{2} (\epsilon + f x)\right]^2 \tan\left[\frac{1}{2} (\epsilon + f x)\right] - \right. \\
& \quad \left. \frac{1}{3 - m - n} (1 - m) (1 - m - n) \operatorname{AppellF1}\left[1 + \frac{1}{2} (1 - m - n), -n, 2 - m, 1 + \frac{1}{2} (3 - m - n), \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (\epsilon + f x)\right]^2, -\tan\left[\frac{1}{2} (\epsilon + f x)\right]^2 \right] \sec\left[\frac{1}{2} (\epsilon + f x)\right]^2 \tan\left[\frac{1}{2} (\epsilon + f x)\right] \right) \right) / \\
& \left( (-1 + m + n) \left( 2 n \operatorname{AppellF1}\left[\frac{1}{2} (3 - m - n), 1 - n, 1 - m, \frac{1}{2} (5 - m - n), \tan\left[\frac{1}{2} (\epsilon + f x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (\epsilon + f x)\right]^2 \right] - 2 (-1 + m) \operatorname{AppellF1}\left[\frac{1}{2} (3 - m - n), -n, 2 - m, \frac{1}{2} (5 - m - n), \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (\epsilon + f x)\right]^2, -\tan\left[\frac{1}{2} (\epsilon + f x)\right]^2 \right] + (-3 + m + n) \operatorname{AppellF1}\left[\frac{1}{2} (1 - m - n), -n, \right. \right. \\
& \quad \left. \left. 1 - m, \frac{1}{2} (3 - m - n), \tan\left[\frac{1}{2} (\epsilon + f x)\right]^2, -\tan\left[\frac{1}{2} (\epsilon + f x)\right]^2 \cot\left[\frac{1}{2} (\epsilon + f x)\right]^2 \right) \right) \right)
\end{aligned}$$

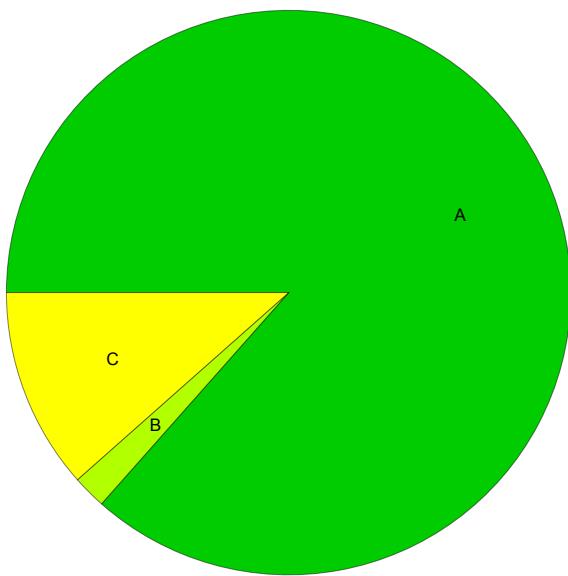




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## Summary of Integration Test Results

52 integration problems



A - 45 optimal antiderivatives

B - 1 more than twice size of optimal antiderivatives

C - 6 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts